Hybrid Sparse Dictionary Construction Using K-SVD and DCT for History Matching by ES-MDA

May 30, 2018

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Contents

Introduction (inverse modeling)

f(m) = d or **d = f(m)**

m: reservoir parameters

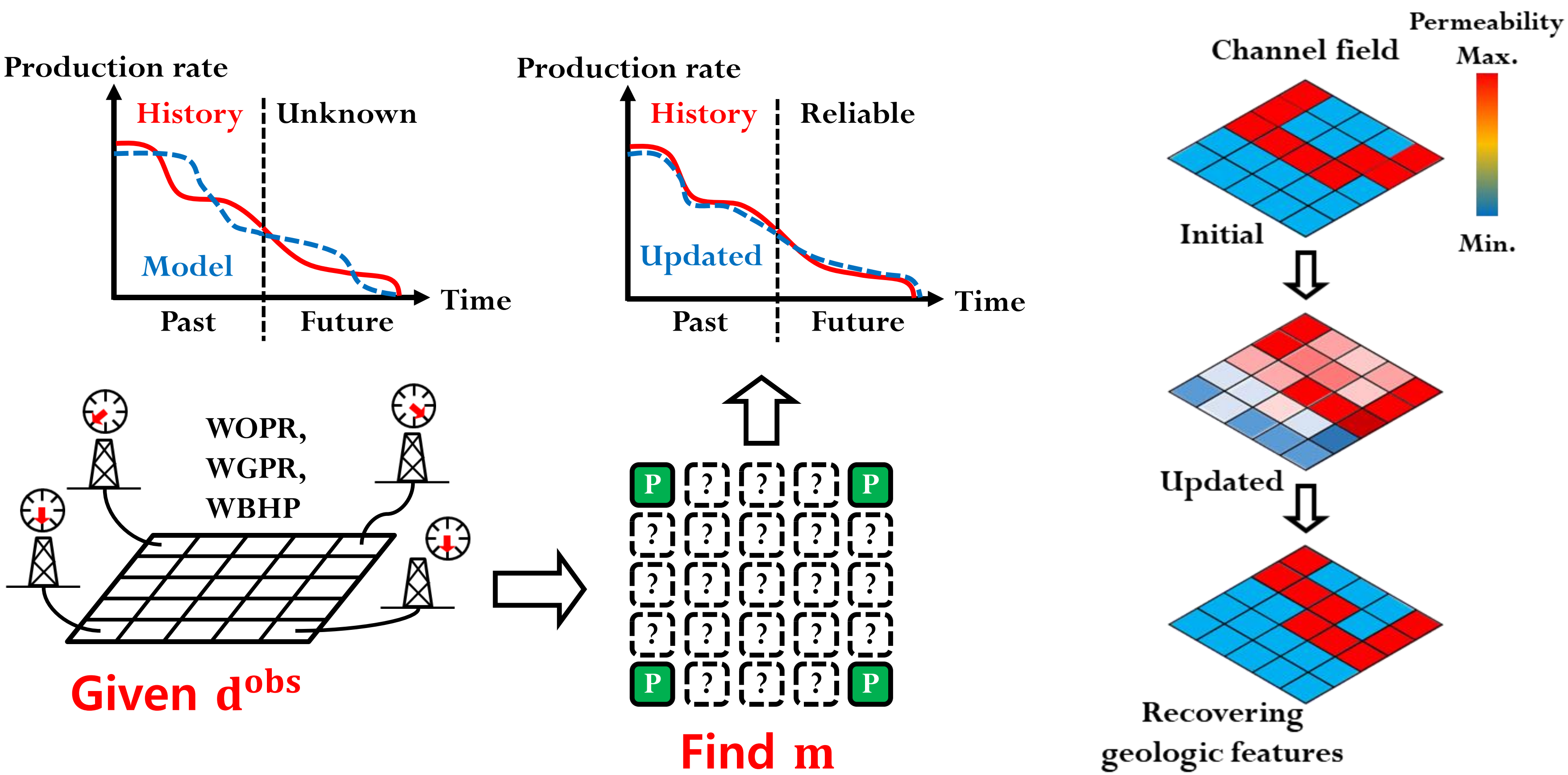
d: simulation responses

f: a reservoir simulator

Limited information

with measurement error and expensive cost

⇨ **Reliable inverse modeling**



Introduction (ensemble-based methods)

Objective function

$$\mathbf{J(m)} = \underbrace{\left(\mathbf{m}-\mathbf{m^b}\right)^{\mathbf{T}}\mathbf{B^{-1}}\left(\mathbf{m}-\mathbf{m^b}\right)}_{J_b\text{, Background term}} + \underbrace{\left(\mathbf{d^{obs}}-\mathbf{d}\right)^{\mathbf{T}}\mathbf{R^{-1}}\left(\mathbf{d^{obs}}-\mathbf{d}\right)}_{J_o\text{, Observation term}} \Rightarrow \nabla\mathbf{J(m)} = \mathbf{0}$$

*Assuming Gaussian dist.

$$\mathbf{m} = \mathbf{m^b} + \mathbf{K}\left(\mathbf{d_{unc}} - \mathbf{d}(\mathbf{m^b})\right)$$

$$\mathbf{K} = \mathbf{C_{md}}\left(\mathbf{C_{dd}} + \alpha\mathbf{C_D}\right)^{-1}$$

(Emerick and Reynolds, 2013; Chen and Oliver, 2013)

m: state vector (model realization)

$\mathbf{m^b}$: state vector before update

B: covariance matrix of $\mathbf{m^b}$

d: simulated response of a state vector

$\mathbf{d^{obs}}$: observation data

$\mathbf{d_{unc}}$: perturbed observed data

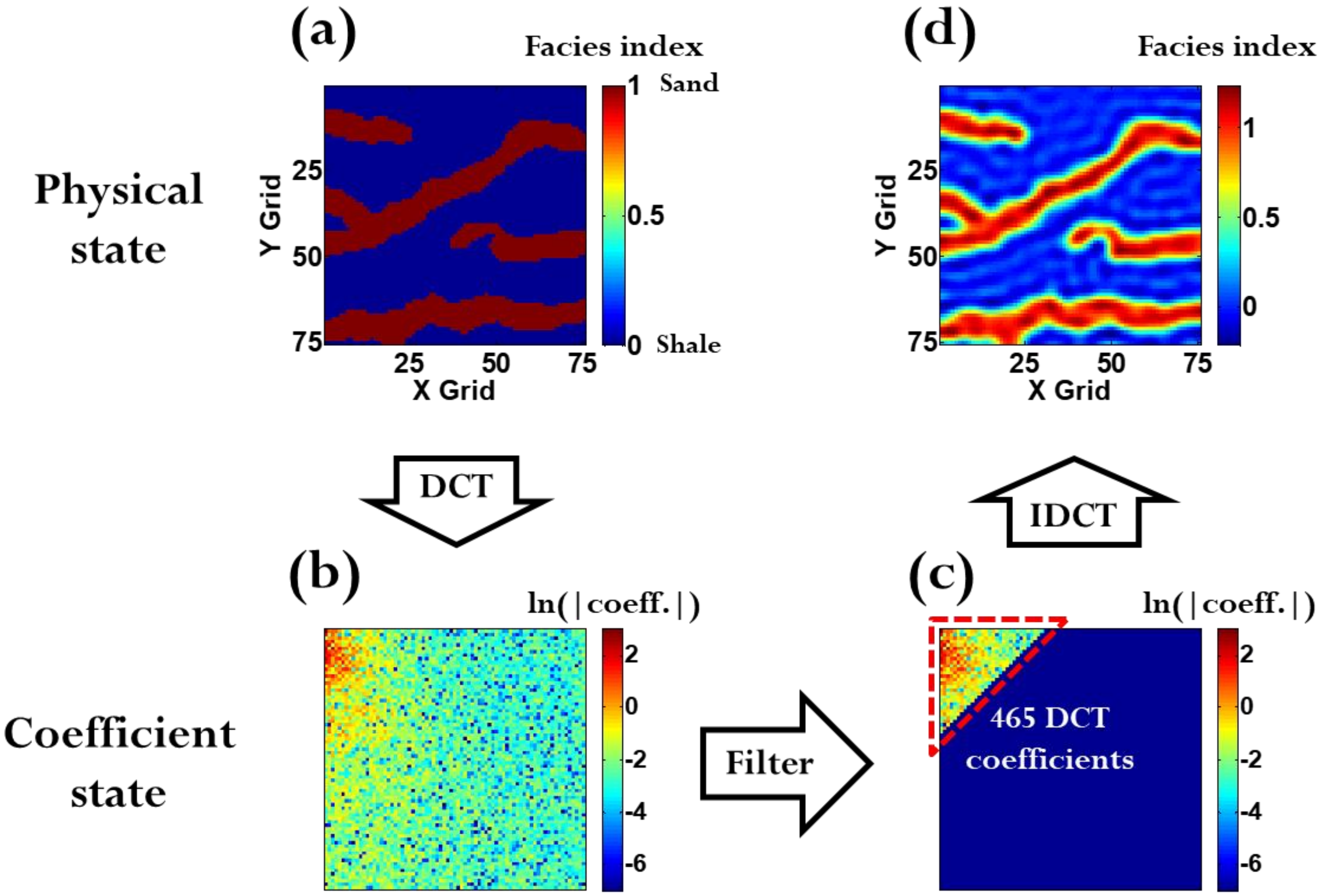
R: covariance matrix of observation error

α: inflating coefficient of $\mathbf{C_D}$

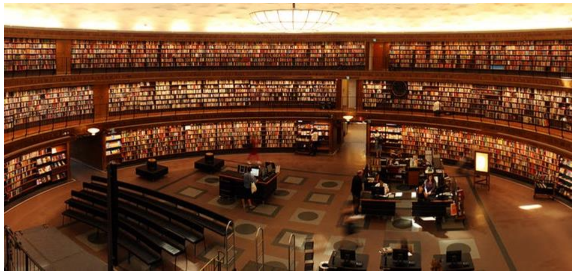
Transformation of parameters of a channel reservoir

- Distribution modification
  - Normal Score Transformation (Shin et al. 2010)
  - Level Set (Lorentzen et al., 2013)
- Image process
  - Discrete Cosine Transform (DCT) (Jafarpour and McLaughlin, 2007)
- Learning algorithm
  - K-Singular Value Decomposition (K-SVD) (Kreutz-Delgado et al., 2003; Aharon et al., 2006)

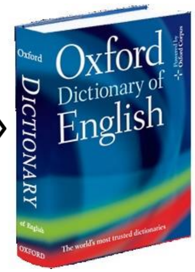
DCT and IDCT application



K-SVD for a geological dictionary



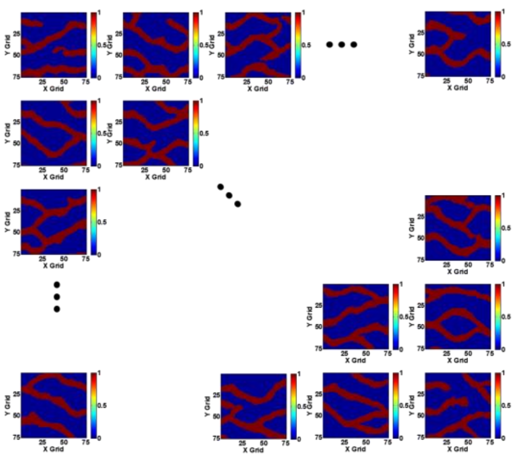
Extract



Words selection

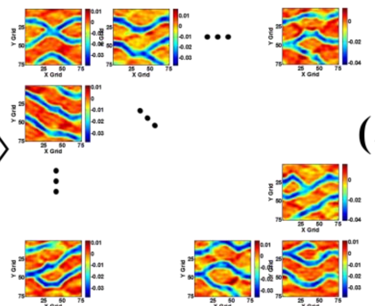
A sentence
"I love cookies"
Or the book
'Romeo & Juliet'
Or even every books
of library

Library (Y, N_{grid} by N_{lib})



K-SVD

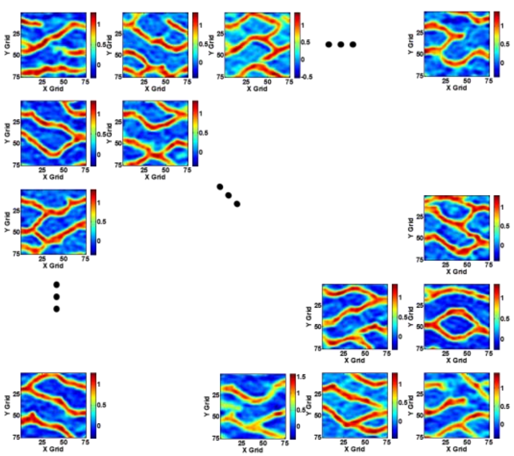
Dictionary (D, N_{grid} by N_{dict})



Weight
coefficients
(N_{dict} by N_{lib})

X

Reconstruction (Y', N_{grid} by N_{lib})



Procedures of K-SVD

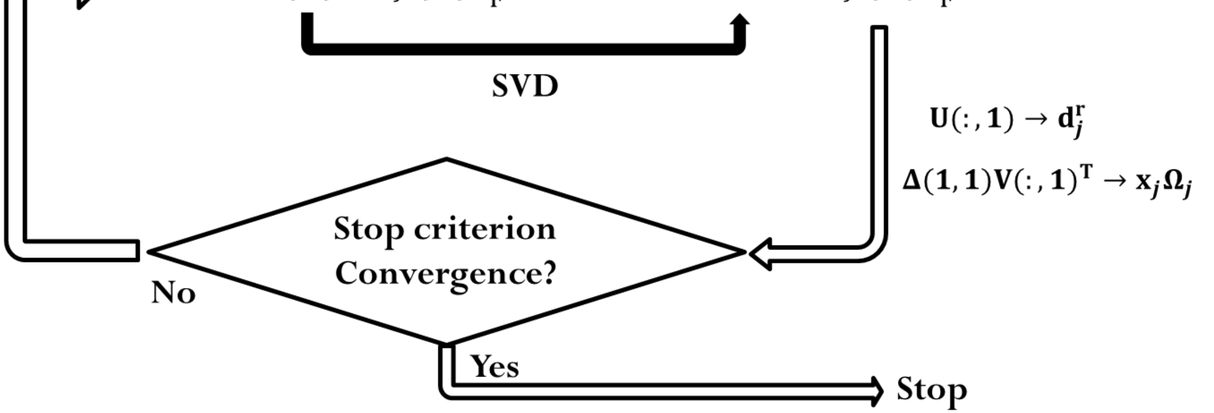
$$\text{minimize } \|\mathrm{Y} - \mathrm{Y}'\|_{\mathrm{F}} = \text{minimize } \left\| \mathrm{Y} - \sum_{i=1}^{N_{\mathrm{dict}}} \mathrm{d}_i^{\mathrm{r}} \mathrm{x}_i \right\|_{\mathrm{F}}$$

⇩

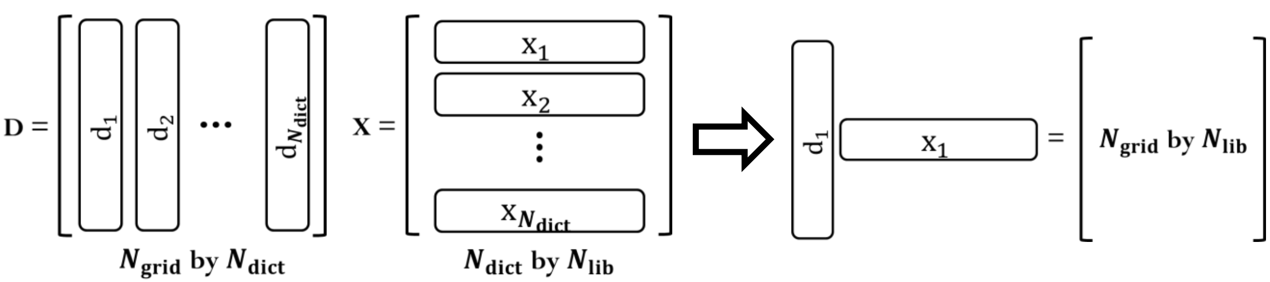
$$\text{minimize } \left\| \left(\mathrm{Y} - \sum_{i \neq j} \mathrm{d}_i^{\mathrm{r}} \mathrm{x}_i \right) - \mathrm{d}_j^{\mathrm{r}} \mathrm{x}_j \right\|_{\mathrm{F}} = \text{minimize } \|\mathrm{E}_j - \mathrm{d}_j^{\mathrm{r}} \mathrm{x}_j\|_{\mathrm{F}} \text{ for } j = 1, \dots, N_{\mathrm{dict}}$$

⇩

$$\text{minimize } \|\mathrm{E}_j \Omega_j - \mathrm{d}_j^{\mathrm{r}} \mathrm{x}_j \Omega_j\|_{\mathrm{F}} \cong \text{minimize } \|\mathrm{U}\Delta\mathrm{V}^{\mathrm{T}} - \mathrm{d}_j^{\mathrm{r}} \mathrm{x}_j \Omega_j\|_{\mathrm{F}} \text{ for } j = 1, \dots, N_{\mathrm{dict}}$$



- **Y: a library**
- **Y′: a reconstructed library**
- **$\mathrm{d}_i^{\mathrm{r}}$: a column vector of dictionary D**
- **x_i: a row vector of weights matrix X**
- **N_{dict}: number of dictionary realizations**
- **E_j: error of all members except $\mathrm{d}_j^{\mathrm{r}} \mathrm{x}_j$**
- **Ω_j: selecting out non-zero components**



$$\overset{\mathbf{x}_j}{(2 \quad 0 \quad 4)} \overset{\Omega_j}{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}} = \overset{\text{Non-zero}}{(2 \quad 4)}$$

Literature review

- Aharon et al. (2006): showed the efficacy of **K-SVD** in image reconstruction.

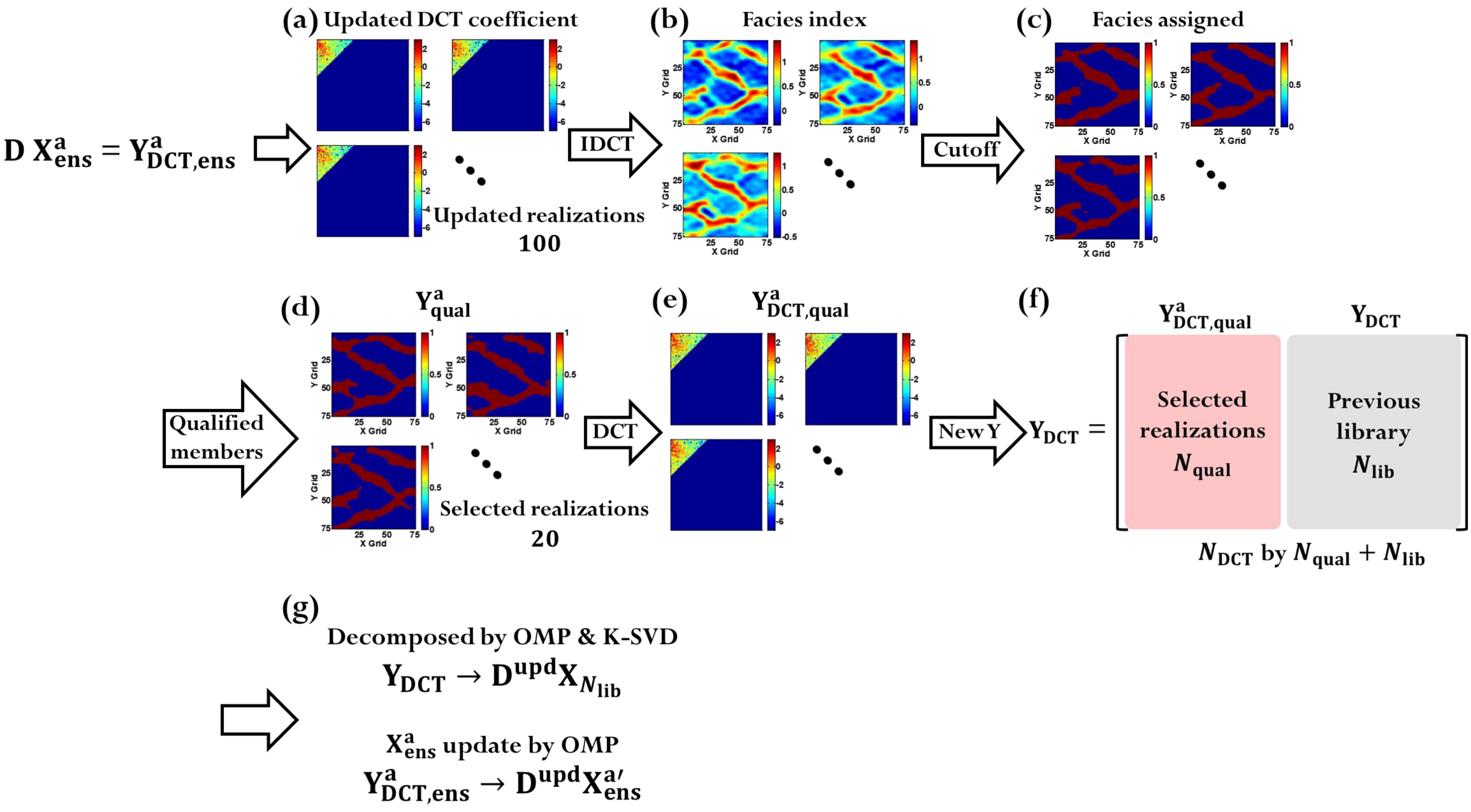
- Li and Jafarpour (2010): extracted essences of geologic features in **DCT** domain.

- Liu and Jafarpour (2013): investigated coupling effects of **DCT** and **K-SVD** for representations of facies connectivity and flow model calibration.

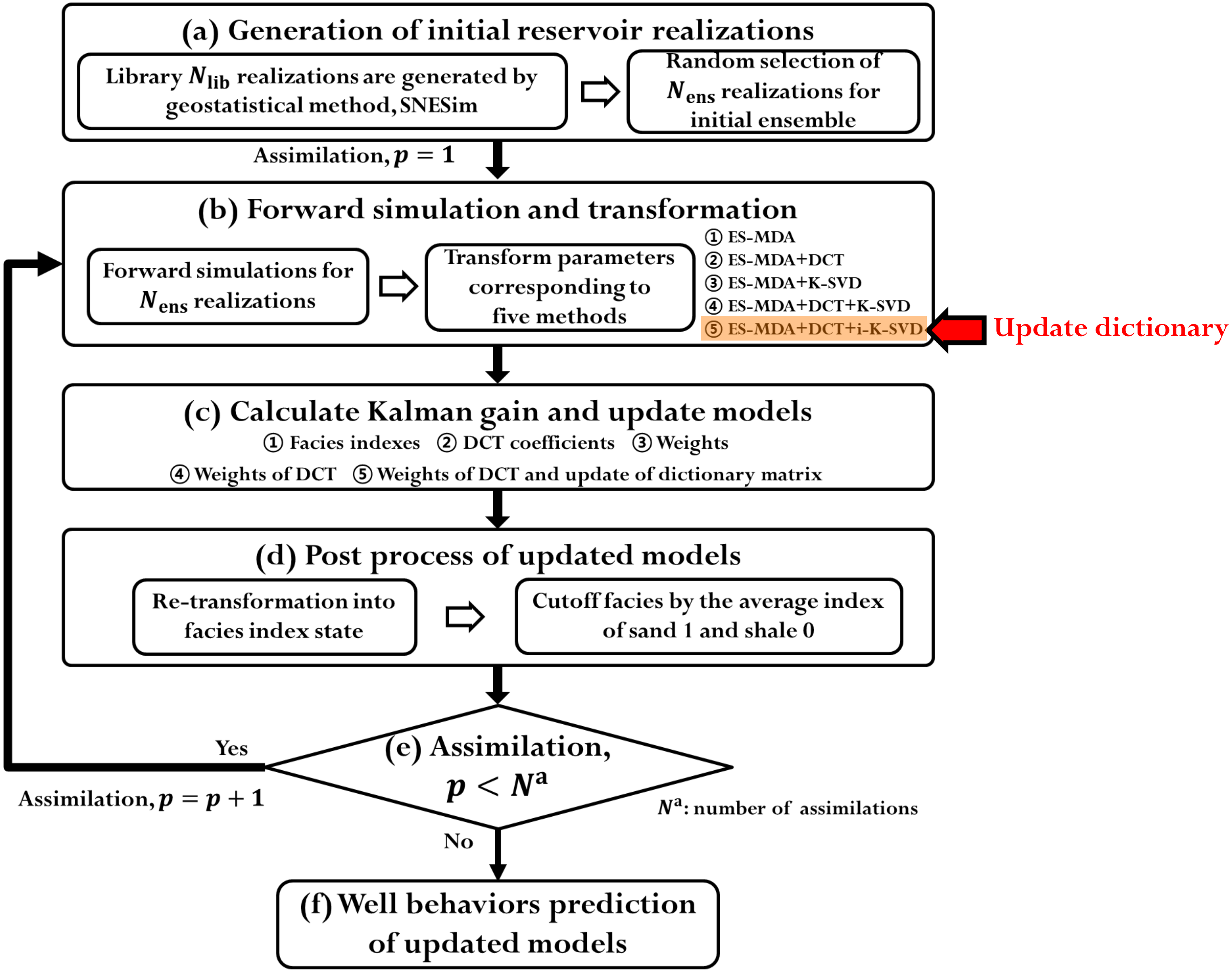
- Sana et al. (2016): built geologic dictionaries from thousands of static reservoir models using **K-SVD** and updated models by **EnKF**

- Proposed method: geologic dictionary update based on **DCT** and **K-SVD** in each assimilation of **ES-MDA**

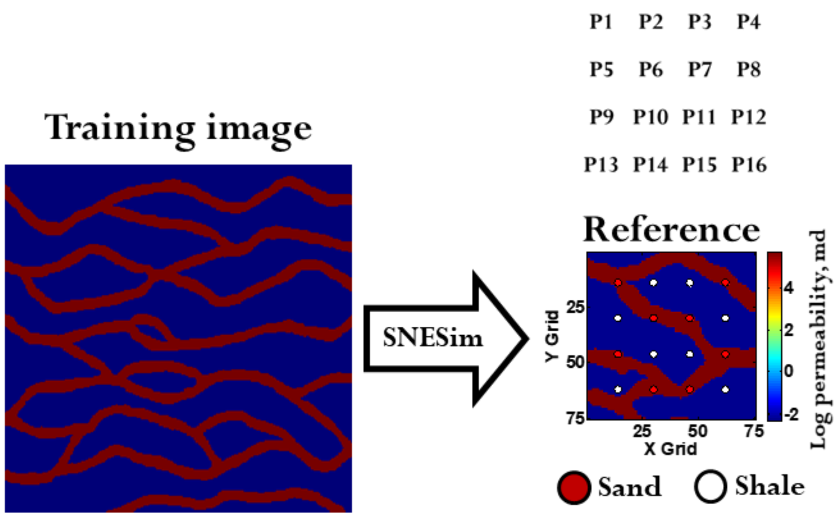
Methodology (Update of a dictionary in ES-MDA)



Methodology (Overall procedure)



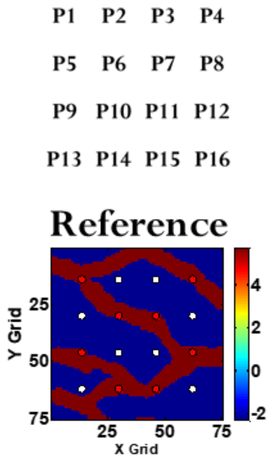
Experimental setting

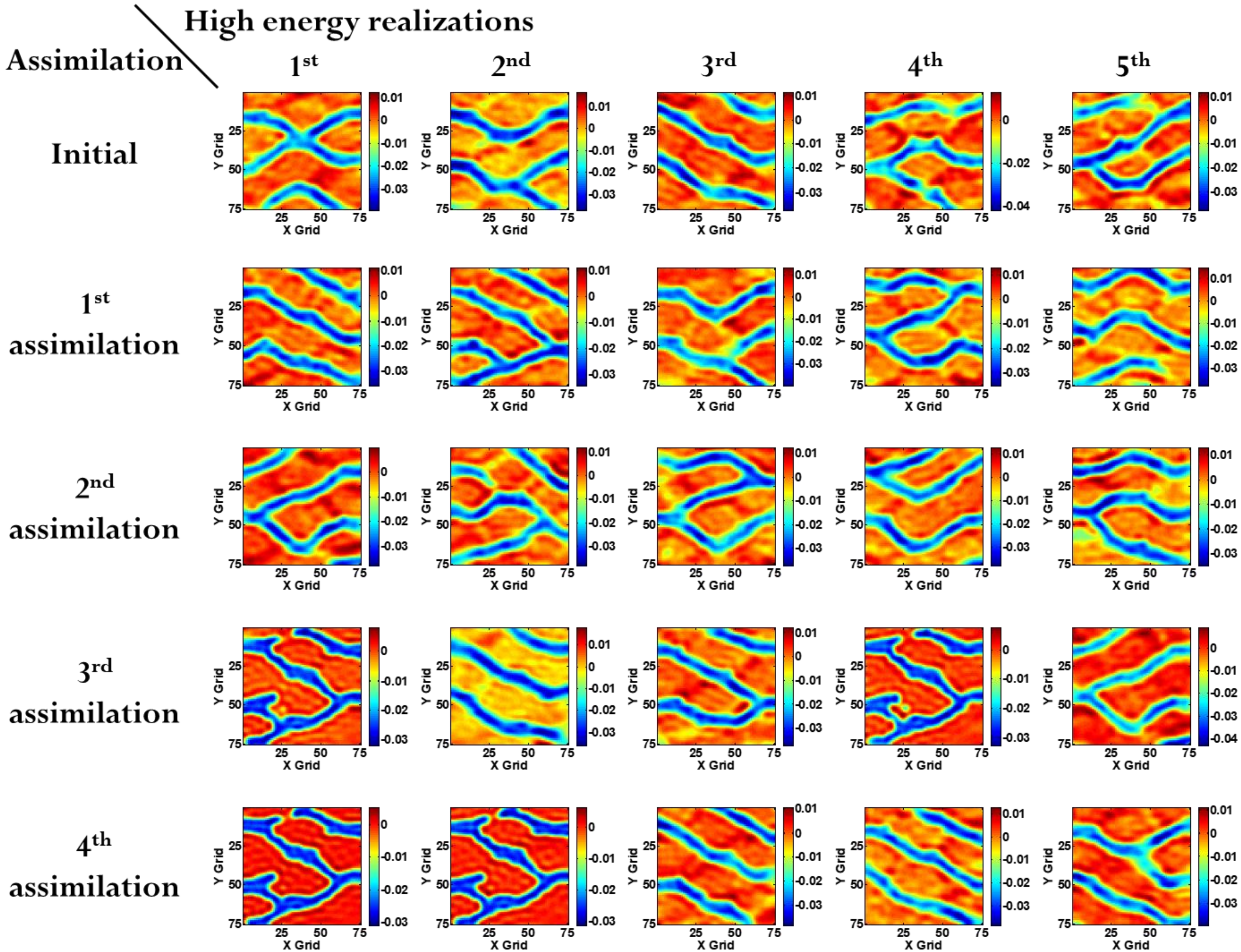


Reservoir parameter	Value
Number of gridblocks in the *x*-direction (N_x) [dimensionless]	75
Number of gridblocks in the *y*-direction (N_y) [dimensionless]	75
Number of gridblocks in the *z*-direction (N_z) [dimensionless]	1
Grid size [ft^3]	200×200×100
Initial gas saturation [fraction]	0.75
Initial water saturation [fraction]	0.25
Initial reservoir pressure [psia]	3,000
Porosity [fraction]	0.2
Permeability of sand facies [md]	300
Permeability of shale facies [md]	0.1

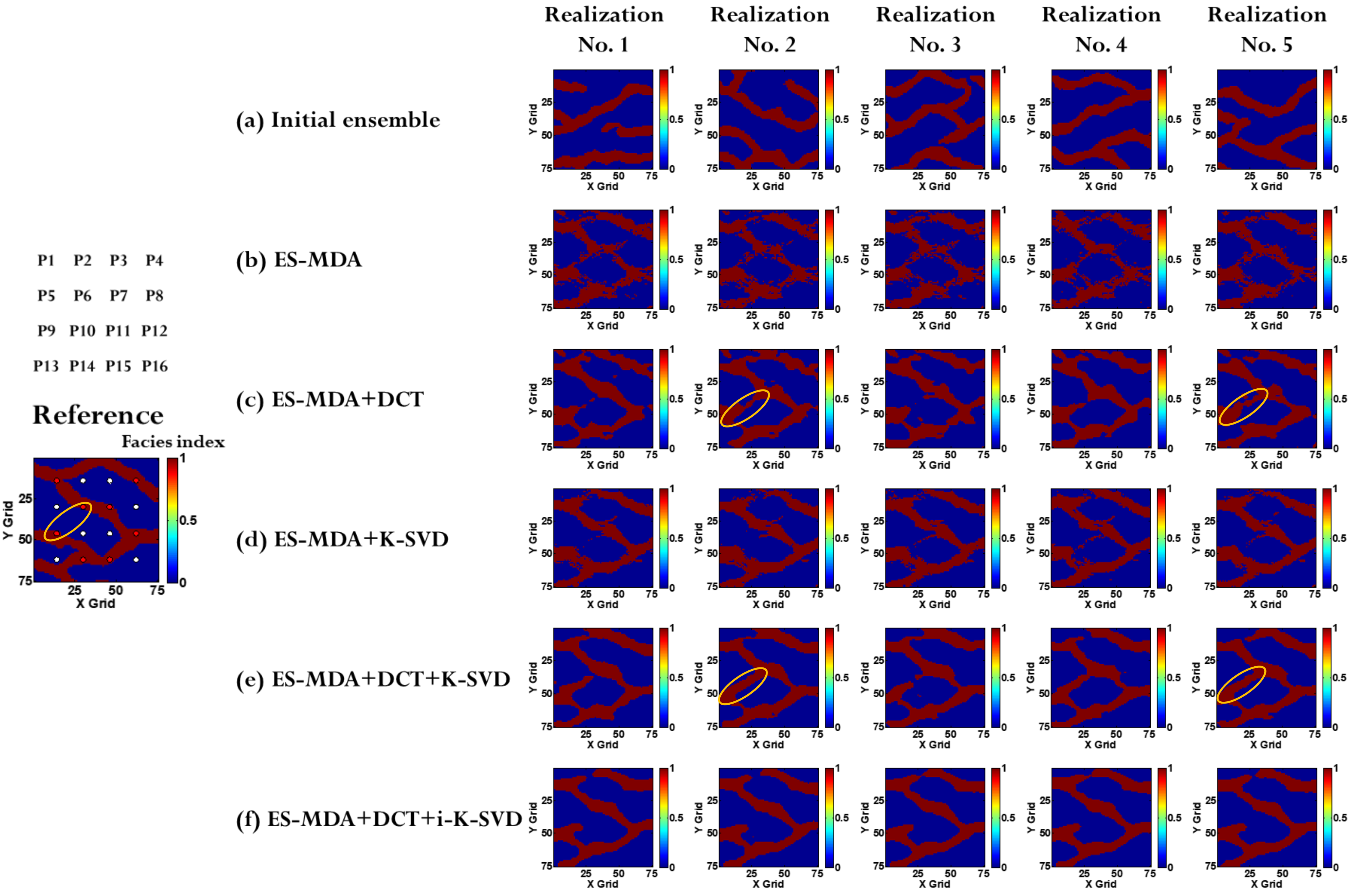
Well parameter	Value
Observed well data	Gas rate and BHP
Maximum well gas production rate [Mscf/day]	15,000
Minimum well BHP [psia]	1,000
Total simulation period [day]	7,000
History matching period [day]	3,500
Prediction period [day]	3,500
Coordinates of well locations in sand facies	(14, 14), (62, 14), (30, 30), (46, 30), (14, 46), (62, 46), (30, 62), (46, 62)
Coordinates of well locations in shale facies	(30, 14), (46, 14), (14, 30), (62, 30), (30, 46), (46, 46), (14, 62), (62, 62)

Dictionaries in each assimilation by the proposed method

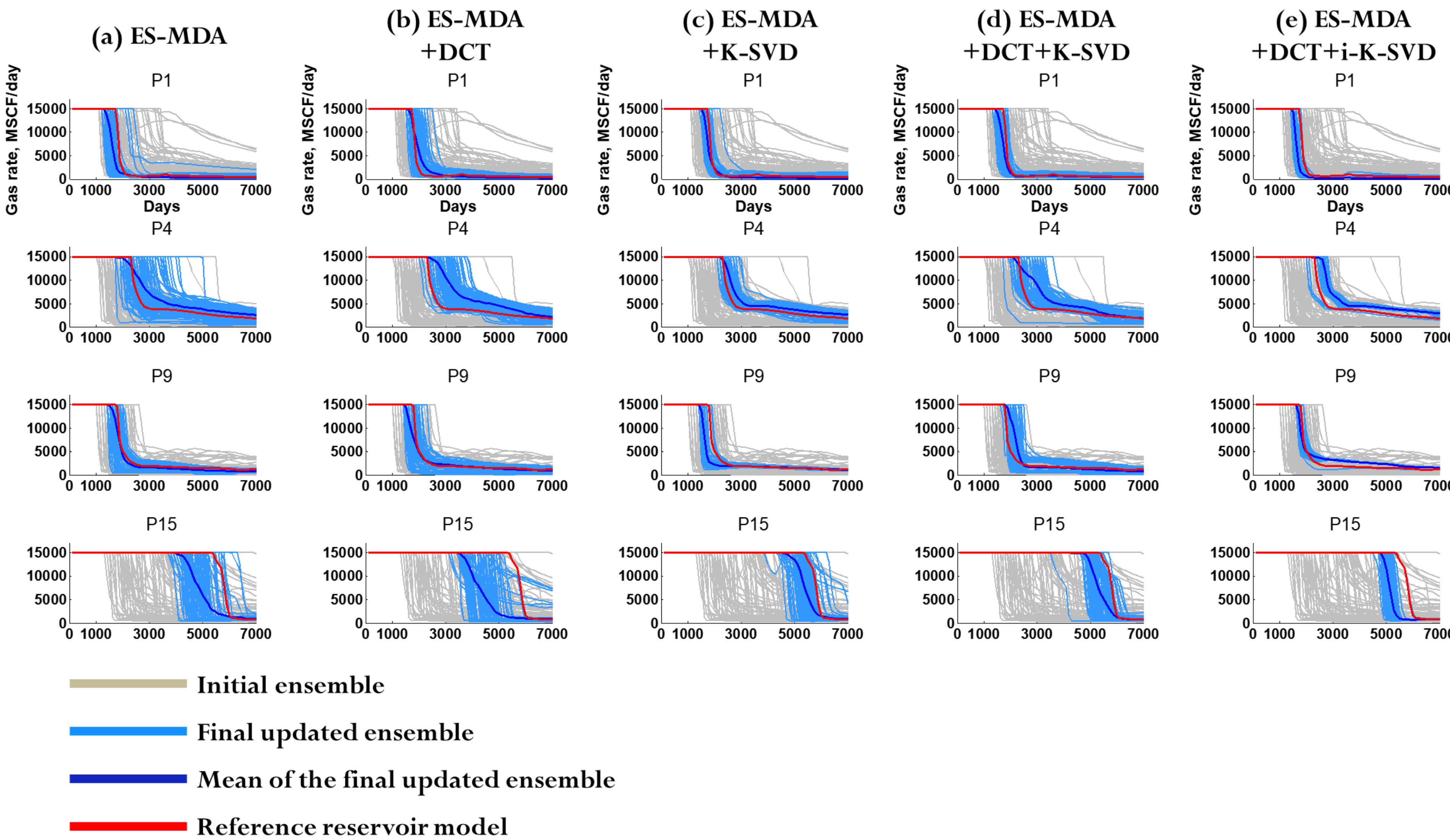




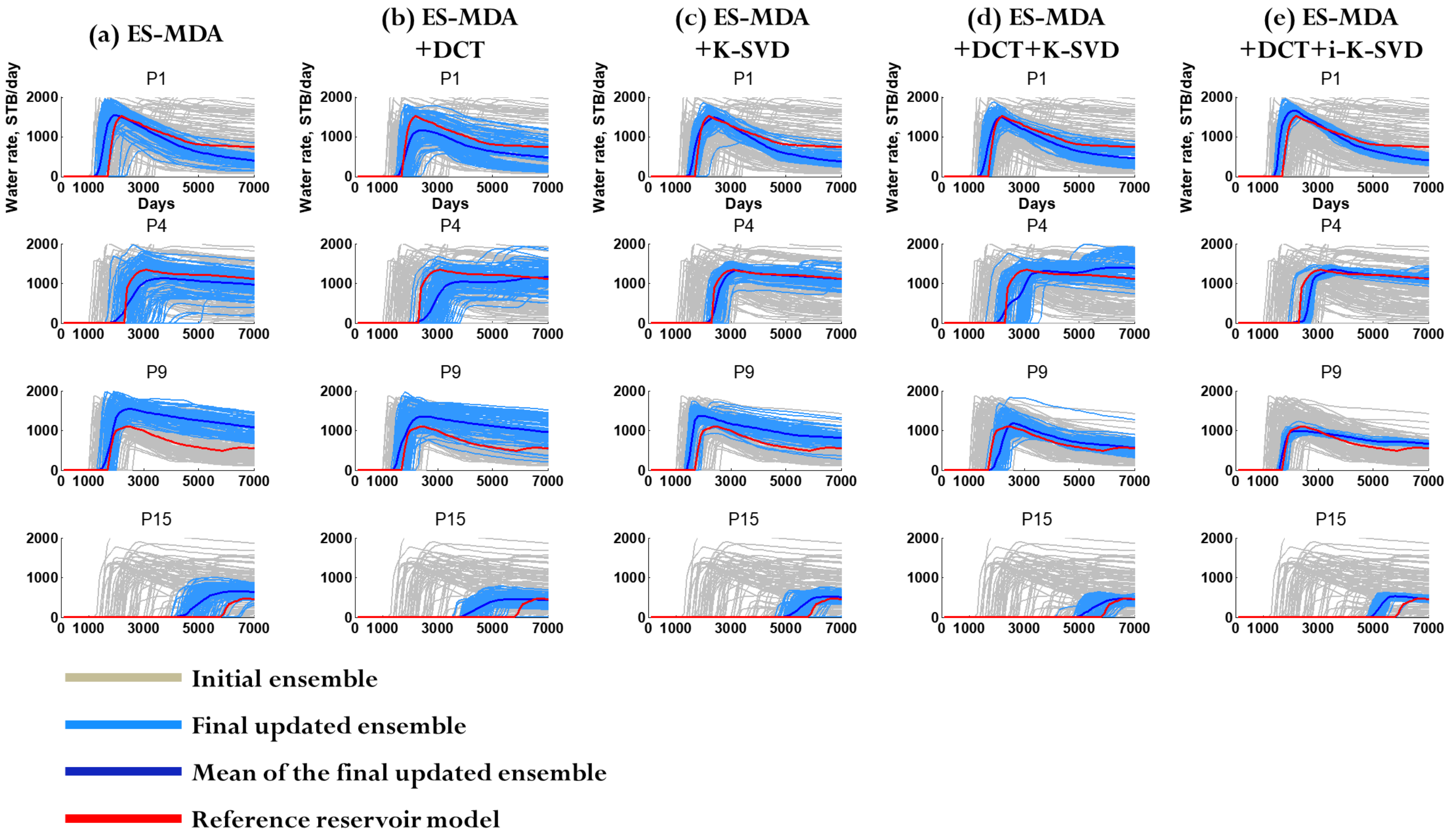
Updated ensemble samples from five methods



Gas rate of the updated ensemble



Water rate of the updated ensemble



Computation time and error of five methods

ES-MDA algorithm	Computational costs [min.]
(a) ES-MDA	0.0
(b) ES-MDA+DCT	0.0
(c) ES-MDA+K-SVD	218.0
(d) ES-MDA+DCT+K-SVD	5.7
(e) ES-MDA+DCT+i-K-SVD	21.4 (=5.7+15.7)

Only for
construction of dictionaries

ES-MDA algorithm	Gas rate (HM)		Water rate (HM)		BHP (HM)	
	μ (%)	σ (%)	μ (%)	σ (%)	μ (%)	σ (%)
ES-MDA	46.36	122.66	15.58	8.38	55.84	27.85
ES-MDA+DCT	194.92	257.50	4.11	2.52	99.01	67.86
ES-MDA+K-SVD	11.81	34.84	5.06	2.94	27.72	29.78
ES-MDA+DCT+K-SVD	5.75	54.73	4.52	4.51	68.11	73.71
ES-MDA+DCT+i-K-SVD	0.99	1.69	5.26	2.40	39.36	16.54
	Gas rate (PD)		Water rate (PD)		BHP (PD)	
	μ (%)	σ (%)	μ (%)	σ (%)	μ (%)	σ (%)
ES-MDA	37.00	48.75	0.53	0.05	27.85	27.70
ES-MDA+DCT	134.11	151.63	0.56	0.04	47.83	30.78
ES-MDA+K-SVD	25.98	16.08	0.10	0.01	8.13	11.79
ES-MDA+DCT+K-SVD	22.91	33.58	0.13	0.01	12.84	14.48
ES-MDA+DCT+i-K-SVD	24.61	13.39	0.19	0.01	13.15	3.54

Only for
8 wells on sand

Initial ensemble
100% error

Conclusions

1. This study proposed a framework of ES-MDA coupled with DCT and K-SVD.

2. This study updated geologic dictionaries with qualified reservoir models considering dynamic observed data during each assimilation of ES-MDA.

3. The proposed method remarkably reduced computational cost and complexity.

4. ES-MDA+DCT+i-K-SVD worked properly and gave overall enhanced performance in terms of channel properties and prediction of productions.

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Q & A

Thank you for your attention

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Acknowledgements

We are thankful for support by KOGAS